

LOCATING CENTERS OF RIGIDITY OF BUILDINGS USING THE STATIC CONDENSATION ALGORITHM

Por Juan Carlos Molano Toro¹

ABSTRACT

This paper presents a new approach for code lateral force analysis of any asymmetric-plan buildings locating centers of rigidity using the static condensation algorithm on the complete stiffness lateral and torsional matrix. A comparative analysis is made through one example using this algorithm.

INTRODUCTION

In the paper from Goel and Chopra(1993) these authors affirm:

- a) "Unlike one-story buildings, there are several difficulties in establishing locations of the CR at various floor levels of a multistory building unless it belongs to a special class known as **proportional buildings** (Cheung and Tso 1986; Hejal and Chopra 1987; Riddell and Vasquez 1984)".
- b) "However, Cheung and Tso (1986) defined the CRs as the set of points located on the building floors through which application of lateral forces would cause no rotation of any of the floors. For a chosen definition of the CRs, their location depends on the heightwise distribution of lateral forces".
- c) "Most commercially available computer programs for structural analysis do not have the capabilities of the matrix calculations required to determine the CRs by available procedures (Cheung and Tso 1986; Hejal and Chopra 1987)".

The numerical approach presented by Molano (2021), supported by Cook (1981) on pages pp 186-187 and Wilson, E. L. (1974) pp. 198-203, will be used here to condense a symmetric and positive lateral-torsional stiffness matrix (referred to an arbitrary origin (0,0) different from the center of mass) to determine the CRs in a quick way. With the help of this numerical algorithm, it will be easy to implement a static procedure in the Code for multi-story buildings with any plan asymmetry.

¹ Civil Engineer, PMP, PMI-SP. juan.molanotoro1@gmail.com, Medellín-Antioquia-Colombia

Computation of Centers of Rigidity Matrix Approach

Consider an N -story building with any arrangement of lateral-load resisting elements connected by rigid diaphragms (Fig. 1).

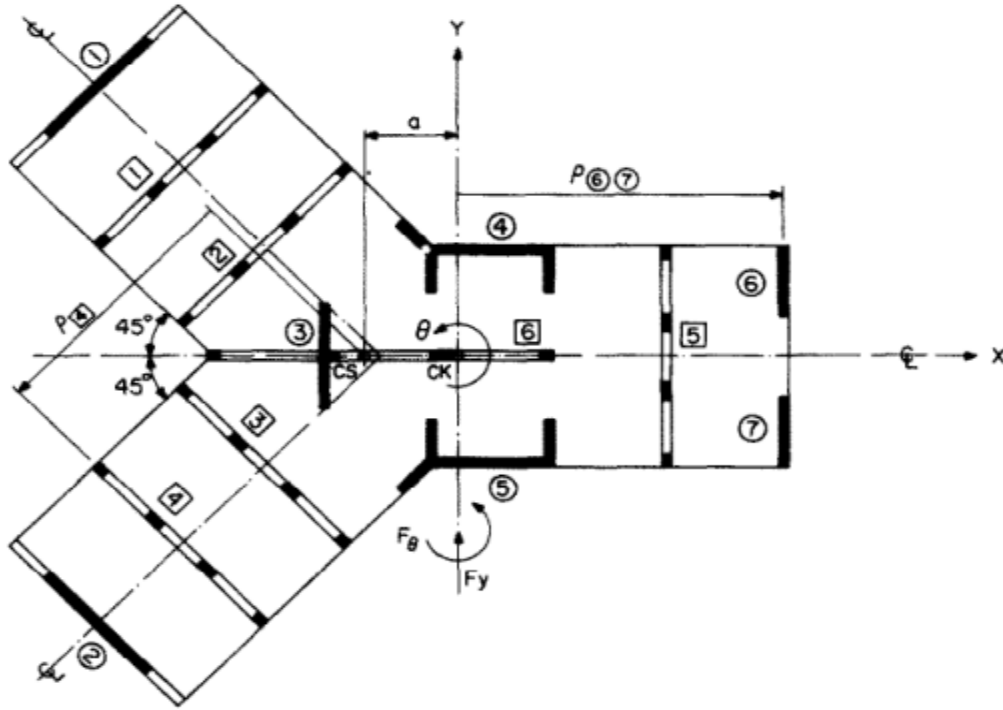


FIG. 1. TYPICAL FLOOR PLAN, ASYMMETRIC WALL-FRAME STRUCTURE from Rutenberg(1979)

For lateral force analysis in the x - and y -directions, the building plan is treated as asymmetric about the y -direction without the loss of generality because building codes require such independent analyses in the x - and y -directions.

The lateral-torsional stiffness matrix for an N -story building has the following format:

$$[K]_{3N \times 3N} = \begin{bmatrix} \text{Kyy} & \text{Kyx} & \text{Ky}\theta \\ \text{SYM} & \text{Kxx} & \text{Kx}\theta \\ & & \text{K}\theta\theta \end{bmatrix} \begin{bmatrix} u_y \\ u_x \\ u_\theta \end{bmatrix} = \begin{bmatrix} F_y \\ F_x \\ F_\theta \end{bmatrix} \quad \text{..... (1)}$$

in which the displacement vectors = $u_y^T = \langle u_{y1}, u_{y2}, \dots, u_{yj}, \dots, u_{yN} \rangle$, $u_x^T = \langle u_{x1}, u_{x2}, \dots, u_{xj}, \dots, u_{xN} \rangle$, y $u_\theta^T = \langle u_{\theta1}, u_{\theta2}, \dots, u_{\theta j}, \dots, u_{\theta N} \rangle$, where u_{yj} = y -lateral displacement, u_{xj} = x -lateral displacement y $u_{\theta j}$ = θ -torsional displacement at a selected reference point on the j th floor; and $F_y^T = \langle F_{y1}, F_{y2}, \dots, F_{yj}, \dots, F_{yN} \rangle$, $F_x^T = \langle F_{x1}, F_{x2}, \dots, F_{xj}, \dots, F_{xN} \rangle$, y $F_\theta^T = \langle F_{\theta1}, F_{\theta2}, \dots, F_{\theta j}, \dots, F_{\theta N} \rangle$, where F_{yj} = lateral force in y -direction, F_{xj} = lateral force in x -direction, y $F_{\theta j}$ = floor torque applied at the selected reference point on

the j th floor. The various submatrices of the stiffness matrix in (1) can be obtained from Winokur and Glück(1968).

ANALYSIS USING THE STATIC CONDENSATION ALGORITHM

In terminology from Cook(1981):

N =number of stories of building,

$NSIZE=3*N$ = number of d.o.f on the uncondensed system,

NUM =the last d.o.f. of $NSIZE$ to condense.

Step 1. The system of equations in (1) is condensed or eliminated with $NUM=2*N$, $F_y^T < [0.0]$, $F_x^T = [0.0]$, $F_\theta^T = [0.0]$, meaning the elimination process of the system of equations is performed from row $i=1$ to row $i=N$.

Step 2. $[F_\theta]$ is obtained using the data from the last N d.o.f.

Step 3. $[X_R]$ is obtained with $[X_R] = \begin{bmatrix} -[F_y]^{-1}*[F_\theta] \\ \end{bmatrix}$ (2)

Step 4. The system of equations in (1) is condensed or eliminated with $NUM=N$, $F_y^T = [0.0]$, $F_x^T < [0.0]$, $F_\theta^T = [0.0]$, meaning the elimination of the system of equations continues from row $i=N+1$ to row $i=2*N$.

Step 5. $[F_\theta]$ is obtained using the data from the last N d.o.f.

Step 6. $[Y_R]$ is obtained with $[Y_R] = \begin{bmatrix} -[F_x]^{-1}*[F_\theta] \\ \end{bmatrix}$ (3)

EXAMPLE USING THE STATIC CONDENSATION ALGORITHM

The example (Goel and Chopra 1993; Tso 1990) is a four-story building having three frames in y -direction and two frames in x -direction. The building is symmetric in x -direction (Fig. 2). The data as follows:

Frames in y -direction: A, B, C.

Frames in x -direction: 1, 2

Torsion in z -direction

$I_{beams} = I_x = I_y = 0.30m^4$ frames A, B, C.

$I_{columns} = I_x = I_y = 0.10m^4$ frame B.

$I_{columns} = I_x = I_y = 0.05m^4$ frames A y C.

($I_{beams} = I_x = I_y = 0.30m^4$ frames 1, 2.

$J_o = I_z = 0.0$ for all beams and columns.

$E = 1.0kN/m^2$, $\nu = \text{poisson} = 0.20$

The columns are assumed to be axially rigid.

The floors weight are 20kN for each of the bottom floors and 10kN for the top two floors.

The base shear V_b for this building example is $= 0.6*60/2 = 18kN$.

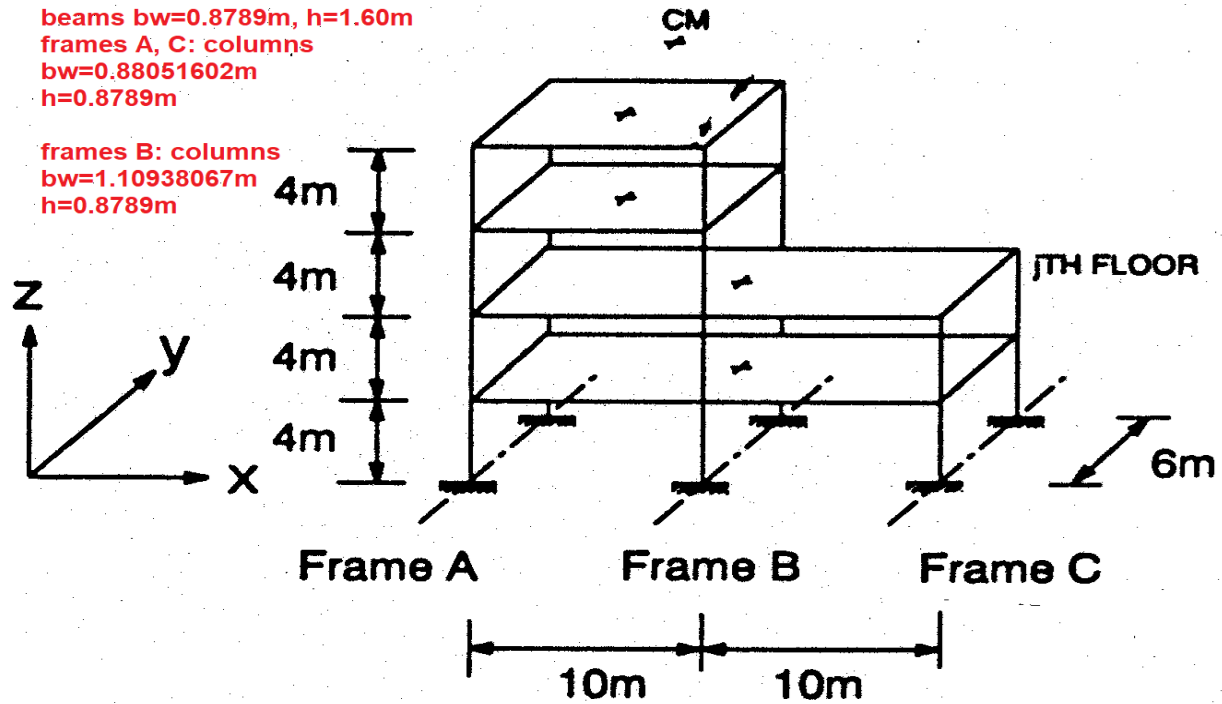


FIG. 2. Example Building, from Goel, R. K and Chopra(1993).

The lateral forces F_{yj} are computed in Table 1 (Goel and Chopra 1993).

TABLE 1. Heightwise Distribution of Lateral Forces

Floor j (1)	Height h_j (2)	Weight w_j (3)	$w_j h_j$ (4)	$F_{yj} = \frac{w_j h_j}{\sum w_j h_j} V_b$ (5)
4	16	10	160	5.54
3	12	10	120	4.15
2	8	20	160	5.54
1	4	20	80	2.77

The total lateral-torsional symmetric stiffness matrix $[K]_{3N \times 3N}$ referred to origin (0,0) arbitrary (does not coincide with the center of mass) is as follows: $[K_{yy}]_{N \times N}$, $[K_{yx}]_{N \times N}$, $[K_{y\theta}]_{N \times N}$, $[K_{xx}]_{N \times N}$, $[K_{x\theta}]_{N \times N}$, $[K_{\theta\theta}]_{N \times N}$; only is shown the upper triangular matrix (Fig 3).

$N=4$ =number of stories of building.

$NSIZE=3 \times N=3 \times 4=12$ = number of d.o.f on the uncondensed system.

NUM =the last d.o.f. of $NSIZE$ to condense.

$NUM=2 \times N=2 \times 4=8$ the last d.o.f. of $NSIZE$ to condense in Step 1.

$NUM=1 \times N=1 \times 4=4$ the last d.o.f. of $NSIZE$ to condense in Step 4.

1	1	1.4046986e-01	2	2	1.1287388e-01	1	2	-7.3100433e-02
3	3	9.6009821e-02	2	3	-5.5419520e-02	1	3	7.4316124e-03
4	4	4.1275118e-02	3	4	-4.7344927e-02	2	4	6.6244239e-03
1	4	-6.1193731e-04	5	5	1.3919538e-01	4	5	0.0000000e+00
3	5	0.0000000e+00	2	5	0.0000000e+00	1	5	0.0000000e+00
6	6	1.0725920e-01	5	6	-7.1733683e-02	4	6	0.0000000e+00
3	6	0.0000000e+00	2	6	0.0000000e+00	1	6	0.0000000e+00
7	7	9.0163246e-02	6	7	-5.3694922e-02	5	7	7.3248111e-03
4	7	0.0000000e+00	3	7	0.0000000e+00	2	7	0.0000000e+00
1	7	0.0000000e+00	8	8	3.5406861e-02	7	8	-4.3139126e-02
6	8	8.4163910e-03	5	8	-7.5120403e-04	4	8	0.0000000e+00
3	8	0.0000000e+00	2	8	0.0000000e+00	1	8	0.0000000e+00
9	9	2.3624289e+01	8	9	2.2536123e-03	7	9	-2.1974439e-02
6	9	2.1520102e-01	5	9	-4.1758609e-01	4	9	-5.0865524e-03
3	9	5.6678720e-02	2	9	-7.1209103e-01	1	9	1.4022357e+00
10	10	1.4385991e+01	9	10	-1.1783752e+01	8	10	-2.5249170e-02
7	10	1.6108479e-01	6	10	-3.2177764e-01	5	10	2.1520102e-01
4	10	4.9848214e-02	3	10	-3.6795151e-01	2	10	9.4116002e-01
1	10	-7.1209103e-01	11	11	7.8513017e+00	10	11	-4.6460228e+00
9	11	6.9863385e-01	8	11	1.2941734e-01	7	11	-2.7048969e-01
6	11	1.6108479e-01	5	11	-2.1974441e-02	4	11	-3.0589554e-01
3	11	6.2283623e-01	2	11	-3.6795151e-01	1	11	5.6678720e-02
12	12	3.2435760e+00	11	12	-3.8354597e+00	10	12	6.4997709e-01
9	12	-6.4387195e-02	8	12	-1.0622059e-01	7	12	1.2941734e-01
6	12	-2.5249170e-02	5	12	2.2536123e-03	4	12	2.6062521e-01
3	12	-3.0589554e-01	2	12	4.9848214e-02	1	12	-5.0865519e-03

Fig. 3 Total Lateral-Torsional Stiffness Matrix [K]_{3N×3N}

Step 1:

P_y[1]= 2.770 (floor 1)
P_y[2]= 5.540 (floor 2)
P_y[3]= 4.150 (floor 3)
P_y[4]= 5.540 (floor 4)
P_x[5]= 0.000 (floor 1)
P_x[6]= 0.000 (floor 2)
P_x[7]= 0.000 (floor 3)
P_x[8]= 0.000 (floor 4)
P_θ[9]= 0.000 (floor 1)
P_θ[10]= 0.000 (floor 2)
P_θ[11]= 0.000 (floor 3)
P_θ[12]= 0.000 (floor 4)

Step 2:

P_y[1]= 19.720 (floor 1)
P_y[2]= 93.295 (floor 2)
P_y[3]= 146.633 (floor 3)
P_y[4]= 1123.947 (floor 4)
P_x[5]= 0.000 (floor 1)
P_x[6]= 0.000 (floor 2)
P_x[7]= 0.000 (floor 3)
P_x[8]= 0.000 (floor 4)
P_θ[9]= -23.682 (floor 1)
P_θ[10]= -97.360 (floor 2)
P_θ[11]= -25.470 (floor 3)
P_θ[12]= -33.238 (floor 4)

Step 3:

X_R[5]= 23.682/2.77= 8.549m (floor 1)
X_R[6]= 97.360/5.54=17.574m (floor 2)
X_R[7]= 25.470/4.15= 6.137m (floor 3)
X_R[8]= 33.238/5.54= 5.999m (floor 4)

Step 4:

P_y[1]= 0.000 (floor 1)
P_y[2]= 0.000 (floor 2)
P_y[3]= 0.000 (floor 3)
P_y[4]= 0.000 (floor 4)
P_x[5]= 2.770 (floor 1)
P_x[6]= 5.540 (floor 2)
P_x[7]= 4.150 (floor 3)
P_x[8]= 5.540 (floor 4)
P_θ[9]= 0.000 (floor 1)
P_θ[10]= 0.000 (floor 2)
P_θ[11]= 0.000 (floor 3)
P_θ[12]= 0.000 (floor 4)

Step 5:

P_y[1]= 0.000 (floor 1)
P_y[2]= 0.000 (floor 2)
P_y[3]= 0.000 (floor 3)
P_y[4]= 0.000 (floor 4)
P_x[5]= 19.900 (floor 1)
P_x[6]= 99.123 (floor 2)
P_x[7]= 164.794 (floor 3)
P_x[8]= 1249.442 (floor 4)
P_θ[9]= 8.310 (floor 1)
P_θ[10]= 16.620 (floor 2)
P_θ[11]= 12.450 (floor 3)
P_θ[12]= 16.620 (floor 4)

Step 6:

Y_R[5]= 8.310/2.77=3.000 (floor 1)
Y_R[6]= 16.620/5.54=3.000 (floor 2)
Y_R[7]= 12.450/4.15=3.000 (floor 3)
Y_R[8]= 16.620/5.54=3.000 (floor 4)

Fig. 4 Steps Using Static Condensation Algorithm

The location of the CRs for all floors in y-direction are shown in column 2 of Table 2 (Goel and Chopra 1993) and have the same results shown in Fig 4 of this paper.

TABLE 2. Location of Centers of Rigidity and Floor Eccentricities for Example Building

Floor <i>j</i> (1)	Location of center of rigidity (m) (2)	Floor eccentricity (m) (3)
4	5.99	0.99
3	6.17	1.17
2	17.55	7.55
1	8.59	-1.41

In the Table 3 are shown shears in columns of frames A, B, C in y-direction using the Standard Approach (CRs) by Molano (1995, pág 792).

TABLE 3. Shears in columns of Frames A, B, C in y-Direction of Example Building by Standard Approach (CRs)

Frame (1)	Floor <i>j</i> (2)	$V_j^{(a)}$ (kN) (3)	$V_j^{(b)}$ (kN) (4)	Design Shear (kN) (5)
A	4	1.612	1.157	1.612
	3	2.814	1.962	2.814
	2	4.229	2.880	4.229
	1	4.517	2.895	4.517
B	4	1.156	1.611	1.611
	3	2.030	2.883	2.883
	2	3.126	2.944	3.126
	1	4.147	4.142	4.147
C	---	---	---	---
	---	---	---	---
	2	0.258	1.789	1.789
	1	0.334	1.960	1.960

$$\alpha = 1.500, \quad \beta = 0.100, \quad \delta = 1.000$$

$$M_{t_i}^{(a)} = F_{ii} [(1.0 - \alpha) CR_i + (\alpha) CM_i - (\beta) b_{j_i}] \quad (4)$$

$$M_{t_i}^{(b)} = F_{ii} [(1.0 - \delta) CR_i + (\delta) CM_i + (\beta) b_{j_i}] \quad (5)$$

$M_{t_i}^{(a)}$, $M_{t_i}^{(b)}$ the floors torque applied at the selected reference point on the i -th floor to obtain the resulting values of the desired response (force or deformation).

In the Table 4 are shown shears in columns of frames A, B, C in y-direction using the New Approach by Molano (1995, pág 793).

**TABLE 4. Shears in columns of Frames A, B, C in y-Direction
of Example Building by New Approach**

Frame (1)	Floor j (2)	V _j ⁽¹⁾ (kN) (3)	V _j ⁽²⁾ (kN) (4)	V _j ⁽³⁾ (kN) (5)	V _j ^(a) (kN) (6)	V _j ^(b) (kN) (7)	Design Shear (kN) (8)
A	4	1.227	1.345	0.186	1.590	1.159	1.590
	3	2.000	2.305	0.342	2.800	1.963	2.800
	2	2.126	3.290	0.410	4.282	2.880	4.282
	1	2.406	3.455	0.559	4.539	2.896	4.539
B	4	1.542	1.424	-0.186	1.179	1.610	1.610
	3	2.845	2.540	-0.342	2.045	2.882	2.882
	2	2.959	3.018	0.074	3.121	2.944	3.121
	1	4.144	4.144	0.002	4.146	4.142	4.146
C	---	---	---	---	---	---	---
	---	---	---	---	---	---	---
	2	2.528	1.305	-0.483	0.211	1.788	1.788
	1	2.448	1.398	-0.561	0.312	1.959	1.959

$$\alpha = 1.500, \quad \beta = 0.100, \quad \delta = 1.000$$

$$r^{(a)} = (1.0 - \alpha)r^{(1)} + (\alpha)r^{(2)} + r^{(3)} \quad (6)$$

$$r^{(b)} = (1.0 - \delta)r^{(1)} + (\delta)r^{(2)} - r^{(3)} \quad (7)$$

$r^{(a)}, r^{(b)}$ the resulting values of the desired response
(force or deformation)

For the former results in Table 3 and Table 4 the mathematical proof is presented in Appendix I.

CONCLUSIONS

A Static Condensation Algorithm has been presented to obtain the CRs for the analysis of multi-story buildings in a quick way. The new approach is complemented and verified by Molano(1995), pages 792-793.

With the help of this numerical algorithm, it will be easy to implement a static procedure in the Code for multi-story buildings with any plan asymmetry, thereby removing one of the major difficulties in buildings analysis.

APPENDIX I. REFERENCES

Cheung, V. W.- T, and Tso, W. K.(1986) "Eccentricity in irregular multistory buildings", *Can. J. Civ. Engrg.*, 13(1), 46-52

Cook et al, (1981), "Concepts and Applications of Finite Element Analysis" 2ed, John Wiley and sons, pp 186-187.

Goel, R. K and Chopra.(1993), A. K. "Seismic Code Analysis of Buildings Without Locating Centers of Rigidity", *J. Struct Engrg.*, ASCE,119(10), 3039-3055

- Goel, R. K and Chopra.(1994), A. K.** "Dual-Level Approach For Seismic Design Of Asymmetric-Pan Building", *J. Struct Engrg.*, ASCE,120(1), 161-179.
- Hejal, R., and Chopra, A. K.(1987).** "Earthquake response of torsionally-coupled buildings", *Report no. UBC/EERC-87/20, Earthquake Engrg. Res. Ctr.*, Univ. Of California, Berkeley, Calif
- Molano T. J. C.(1995)** Discussion on "Seismic Code Analysis of Buildings Without Locating Centers of Rigidity", *J. Struct Engrg.*, ASCE,121(04), 792-793
- Molano T. J. C.(2003)** "Análisis Sísmico Estático de Edificios sin localizar Centros de Rigidez", *XV JORNADAS ESTRUCTURALES DE LA INGENIERIA DE COLOMBIA-SCI*, Bogotá, Colombia.
- Molano T. J. C.(2021)** "Condensación Estática: Una Nueva Aproximación", *XIX JORNADAS GEOTÉCNICAS-XXII JORNADAS ESTRUCTURALES DE LA INGENIERIA DE COLOMBIA-SCI*, Bogotá, Colombia.
- Riddell, R. and Vazquez, J.(1984).** "Existence of centers of resistance and torsional uncoupling of earthquake response of buildings", *Proc. 8th World Conf. On Earthquake Engrg.*, 4,187-194.
- Rutenberg A.(1979).** "Plane Frame Analysis of Laterally Loaded Asymmetric Building-An Uncoupled Solution", *Computers and Structures*, Vol 10, pp.553-555
- Wilson, E. L. (1974),** "The Static Condensation Algorithm", *International Journal for Numerical Methods in Engineering*, pp. 198-203
- Winokur A., and Glück, J.(1968)** "Lateral Loads in Asymmetric Multistory Structures" , *J. Struct Div.*, ASCE,94, ST3, 645-656

ANÁLISIS SIN USAR CENTROS DE RIGIDEZ

ANÁLISIS USANDO CENTROS DE RIGIDEZ (CR's)

(24')

$$M_t^{(1)} = F_{ii}((1.0 - \alpha)CR_i + (\alpha)CM_i - (\beta)b_{ji})$$

(23')

$$M_t^{(2)} = F_{ii}((1.0 - \delta)CR_i + (\delta)CM_i + (\beta)b_{ji})$$

TABLA 3.1. DATOS BASICOS PARA DIRECCION Y

Yr [1]	=	8.5493e+00	Yc [1]	=	1.0000e+01	b _j y [1]	=	2.0000e+01
Yr [2]	=	1.7574e+01	Yc [2]	=	1.0000e+01	b _j y [2]	=	2.0000e+01
Yr [3]	=	6.1373e+00	Yc [3]	=	5.0000e+00	b _j y [3]	=	1.0000e+01
Yr [4]	=	5.9996e+00	Yc [4]	=	5.0000e+00	b _j y [4]	=	1.0000e+01
Fyy [1]	=	2.7700e+00	Mt ⁽¹⁾ [1]	=	2.4169e+01	Mt ⁽²⁾ [1]	=	3.3240e+01
Fyy [2]	=	5.5400e+00	Mt ⁽¹⁾ [2]	=	2.3340e+01	Mt ⁽²⁾ [2]	=	6.6480e+01
Fyy [3]	=	4.1500e+00	Mt ⁽¹⁾ [3]	=	1.4240e+01	Mt ⁽²⁾ [3]	=	2.4900e+01
Fyy [4]	=	5.5400e+00	Mt ⁽¹⁾ [4]	=	1.9391e+01	Mt ⁽²⁾ [4]	=	3.3240e+01

TABLA 3.2. DATOS BASICOS PARA DIRECCION X

Xr [1]	=	3.0000e+00	Xc [1]	=	3.0000e+00	b _j x [1]	=	6.0000e+00
Xr [2]	=	3.0000e+00	Xc [2]	=	3.0000e+00	b _j x [2]	=	6.0000e+00
Xr [3]	=	3.0000e+00	Xc [3]	=	3.0000e+00	b _j x [3]	=	6.0000e+00
Xr [4]	=	3.0000e+00	Xc [4]	=	3.0000e+00	b _j x [4]	=	6.0000e+00
Fxx [1]	=	2.7700e+00	Mt ⁽¹⁾ [1]	=	-6.6480e+00	Mt ⁽²⁾ [1]	=	-9.9720e+00
Fxx [2]	=	5.5400e+00	Mt ⁽¹⁾ [2]	=	-1.3296e+01	Mt ⁽²⁾ [2]	=	-1.9944e+01
Fxx [3]	=	4.1500e+00	Mt ⁽¹⁾ [3]	=	-9.9600e+00	Mt ⁽²⁾ [3]	=	-1.4940e+01
Fxx [4]	=	5.5400e+00	Mt ⁽¹⁾ [4]	=	-1.3296e+01	Mt ⁽²⁾ [4]	=	-1.9944e+01

TABLA 3.3. VECTORES DE FUERZAS EN ORIGEN (0,0)

DOF	Py1	Py2	Px1	Px2	Py	Px
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	2.7700e+00	2.7700e+00	0.0000e+00	0.0000e+00	2.7700e+00	0.0000e+00
2	5.5400e+00	5.5400e+00	0.0000e+00	0.0000e+00	5.5400e+00	0.0000e+00
3	4.1500e+00	4.1500e+00	0.0000e+00	0.0000e+00	4.1500e+00	0.0000e+00
4	5.5400e+00	5.5400e+00	0.0000e+00	0.0000e+00	5.5400e+00	0.0000e+00
5	0.0000e+00	0.0000e+00	2.7700e+00	0.0000e+00	0.0000e+00	2.7700e+00
6	0.0000e+00	0.0000e+00	5.5400e+00	0.0000e+00	0.0000e+00	5.5400e+00
7	0.0000e+00	0.0000e+00	4.1500e+00	0.0000e+00	0.0000e+00	4.1500e+00
8	0.0000e+00	0.0000e+00	5.5400e+00	0.0000e+00	0.0000e+00	5.5400e+00
9	2.4169e+01	3.3240e+01	-6.6480e+00	-9.9720e+00	2.3682e+01	-8.3100e+00
10	2.3340e+01	6.6480e+01	-1.3296e+01	-1.9944e+01	9.7360e+01	-1.6620e+01
11	1.4240e+01	2.4900e+01	-9.9600e+00	-1.4940e+01	2.5470e+01	-1.2450e+01
12	1.9391e+01	3.3240e+01	-1.3296e+01	-1.9944e+01	3.3238e+01	-1.6620e+01

columnas (6), (7) de esta tabla: Fuerzas sobre CR de cada piso trasladadas al origen (0,0)

TABLA 3.4. VECTORES DE DESPLAZAMIENTOS EN ORIGEN (0,0)

DOF	Dy1	Dy2	Dx1	Dx2	Dy	Dx
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	5.9442e+02	3.7915e+02	-3.0223e+01	3.0223e+01	3.1425e+02	0.0000e+00
2	1.2507e+03	8.1426e+02	-6.0813e+01	6.0813e+01	6.5315e+02	0.0000e+00
3	1.7100e+03	1.1241e+03	-9.4066e+01	9.4066e+01	9.5016e+02	0.0000e+00
4	1.9697e+03	1.2986e+03	-1.1435e+02	1.1435e+02	1.1239e+03	0.0000e+00
5	-8.4040e+01	-1.9472e+01	3.3441e+02	3.1628e+02	0.0000e+00	3.2535e+02
6	-1.8011e+02	-4.8384e+01	7.0270e+02	6.6585e+02	0.0000e+00	6.8428e+02
7	-2.5279e+02	-5.2846e+01	1.0588e+03	9.9099e+02	0.0000e+00	1.0249e+03
8	-2.8013e+01	-6.4905e+00	3.0212e+00	-3.0212e+00	0.0000e+00	0.0000e+00
9	-6.0037e+01	-1.6128e+01	6.1421e+00	-6.1421e+00	0.0000e+00	0.0000e+00
10	-8.4263e+01	-1.7615e+01	1.1307e+01	-1.1307e+01	0.0000e+00	0.0000e+00
11	-9.8396e+01	-1.7731e+01	1.4651e+01	-1.4651e+01	0.0000e+00	0.0000e+00

columnas (6), (7) de esta tabla: Fuerzas sobre CR de cada piso trasladadas al origen (0,0)

TABLA 4.1. VECTORES DE FUERZAS EN ORIGEN (0,0)

DOF	Py1	Py2	Py3	Px1	Px2	Px3
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	2.7700e+00	2.7700e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
2	5.5400e+00	5.5400e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
3	4.1500e+00	4.1500e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
4	5.5400e+00	5.5400e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
5	0.0000e+00	0.0000e+00	0.0000e+00	2.7700e+00	2.7700e+00	0.0000e+00
6	0.0000e+00	0.0000e+00	0.0000e+00	5.5400e+00	5.5400e+00	0.0000e+00
7	0.0000e+00	0.0000e+00	0.0000e+00	4.1500e+00	4.1500e+00	0.0000e+00
8	0.0000e+00	0.0000e+00	0.0000e+00	5.5400e+00	5.5400e+00	0.0000e+00
9	restrained	2.7700e+01	-5.5400e+00	restrained	-8.3100e+00	3.3240e+00
10	restrained	5.5400e+01	-1.1080e+01	restrained	-1.6620e+01	2.4900e+00
11	restrained	2.0750e+01	-4.1500e+00	restrained	-1.2450e+01	3.3240e+00
12	restrained	2.7700e+01	-5.5400e+00	restrained	-1.6620e+01	1.6620e+00

TABLA 4.2. VECTORES DE DESPLAZAMIENTOS EN ORIGEN (0,0)

DOF	r1y	r2y	r3y	r1x	r2x	r3x
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	3.1425e+02	4.5227e+02	7.3127e+01	0.0000e+00	7.6586e-06	-3.0223e+01
2	6.5315e+02	9.5661e+02	1.4235e+02	0.0000e+00	3.4996e-05	-6.0813e+01
3	9.5016e+02	1.3237e+03	1.9959e+02	0.0000e+00	1.5161e-04	-9.4066e+01
4	1.1239e+03	1.5321e+03	2.3348e+02	0.0000e+00	5.3547e-05	-1.1435e+02
5	0.0000e+00	-4.1405e+01	-2.1933e+01	3.2535e+02	3.2535e+02	9.0636e+00
6	0.0000e+00	-9.1398e+01	-4.3014e+01	6.8428e+02	6.8428e+02	1.8426e+01
7	0.0000e+00	-1.2225e+02	-6.9408e+01	1.0249e+03	1.0249e+03	3.3920e+01
8	0.0000e+00	-1.3935e+02	-8.6159e+01	1.2494e+03	1.2494e+03	4.3954e+01
9	0.0000e+00	-1.3802e+01	-7.3110e+00	0.0000e+00	3.0212e+00	3.0212e+00
10	0.0000e+00	-3.0466e+01	-1.4338e+01	0.0000e+00	-3.7109e-06	6.1421e+00
11	0.0000e+00	-4.0751e+01	-2.3136e+01	0.0000e+00	-2.1546e-05	1.1307e+01
12	0.0000e+00	-4.6451e+01	-2.8720e+01	0.0000e+00	-6.4041e-06	1.4651e+01

TABLA 4.3. VECTORES DE DESPLAZAMIENTOS EN ORIGEN (0,0)

$$D1 = (1.0 - \alpha)r1 + (\alpha)r2 + r3 \quad (23)$$

$$D2 = (1.0 - \delta)r1 + (\delta)r2 - r3 \quad (24)$$

DOF	Dy1	Dy2	Dx1	Dx2
(1)	(2)	(3)	(4)	(5)
1	5.9442e+02	3.7915e+02	-3.0223e+01	3.0223e+01
2	1.2507e+03	8.1426e+02	-6.0813e+01	6.0813e+01
3	1.7100e+03	1.1241e+03	-9.4066e+01	9.4066e+01
4	1.9697e+03	1.2986e+03	-1.1435e+02	1.1435e+02
5	-8.4040e+01	-1.9472e+01	3.3441e+02	3.1628e+02
6	-1.8011e+02	-4.8384e+01	7.0270e+02	6.6585e+02
7	-2.5279e+02	-5.2846e+01	1.0588e+03	9.9099e+02
8	-2.8013e+01	-6.4905e+00	1.2934e+03	1.2055e+03
9	-2.8013e+01	-6.4905e+00	3.0212e+00	-3.0212e+00
10	-6.0037e+01	-1.6128e+01	6.1421e+00	-6.1421e+00
11	-8.4263e+01	-1.7615e+01	1.1307e+01	-1.1307e+01
12	-9.8396e+01	-1.7731e+01	1.4651e+01	-1.4651e+01